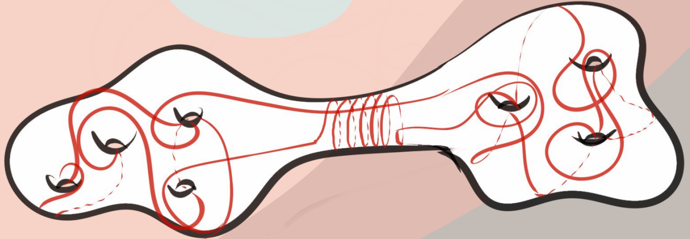
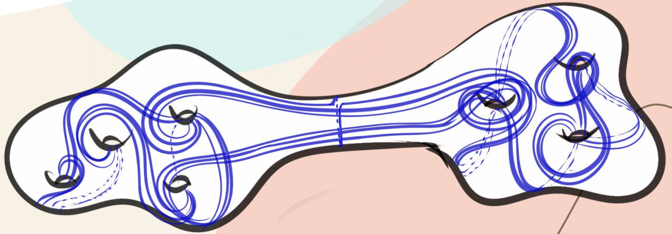
Are these curves different?

Sayantika Mondal
Graduate Center, CUNY





Filling Curves

Cut along curve —--> Disks and annuli

Alternate Characterization:

A closed curve on a surface is said to be filling if it intersects every essential simple, non-peripheral closed curve on the surface.

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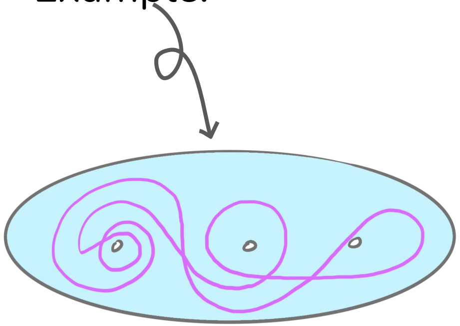
Example:



Topological Types:

Two curves are said to be of the same topological type if there is a mapping class group element (think Dehn twists!) taking one to the other.

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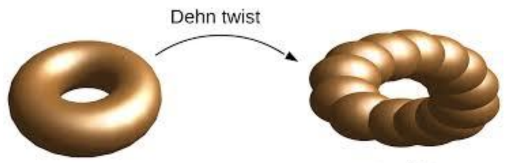

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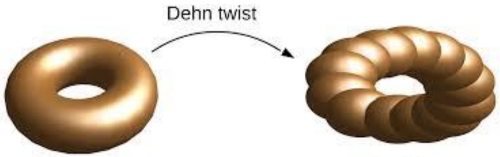

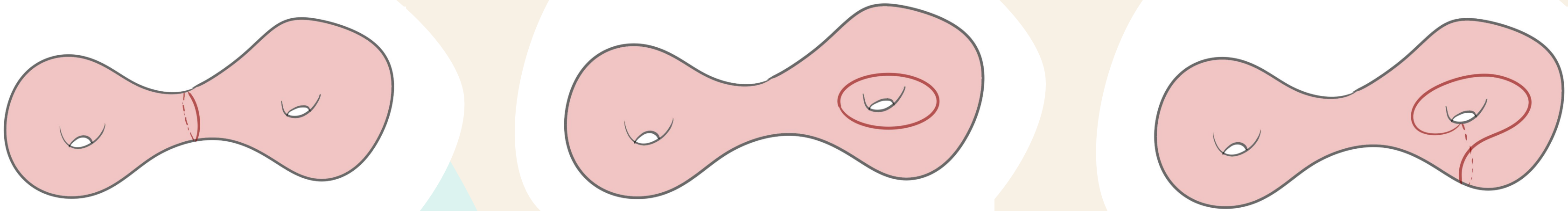

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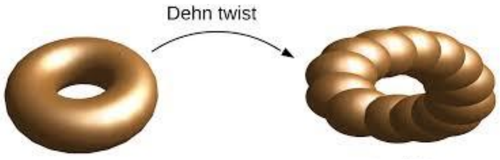

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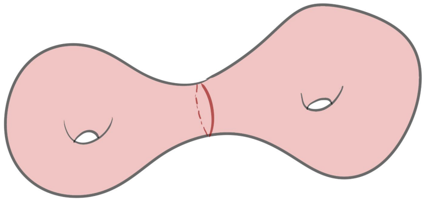

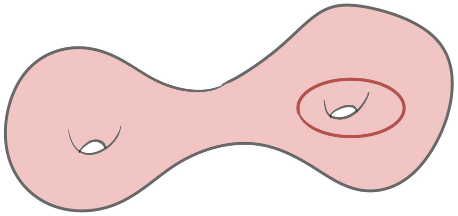

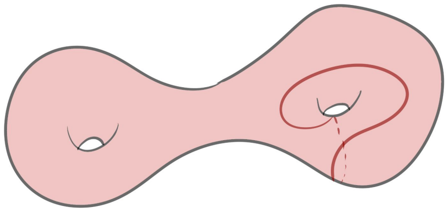

Different

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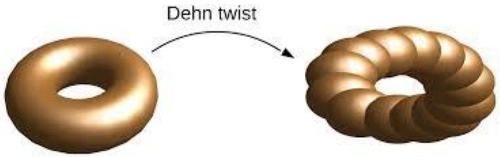

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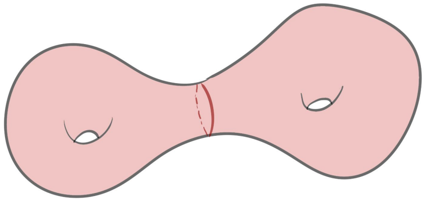

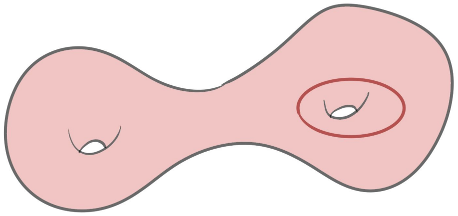

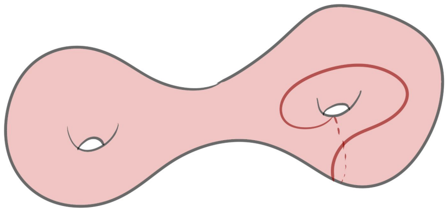

Different

Same

A first example...

Minimal filling curve

Filling curve with the minimum self intersection number on a given surface.

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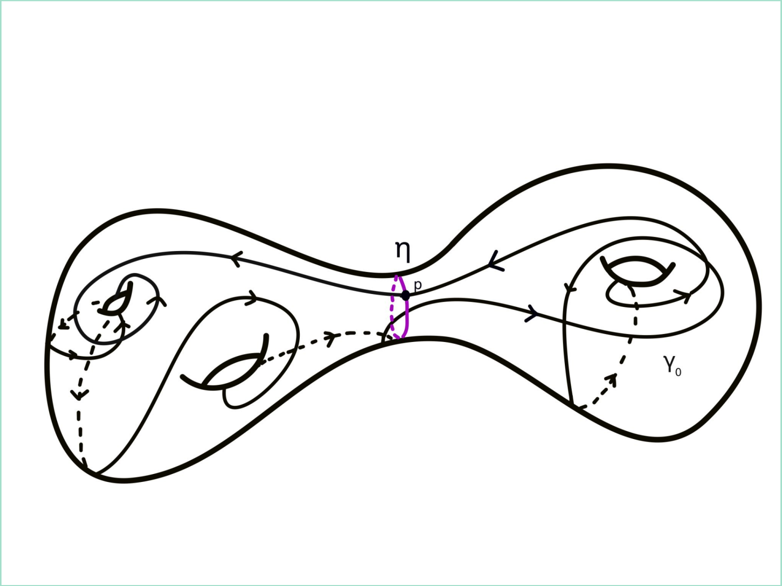
Minimal filling curve

Filling curve with the minimum self intersection number on a given surface.

- Exists!
- This is sort of the simplest example on a surface.

Building filling curves (an infinite family)

Start with a (minimal) filling curve γ_0



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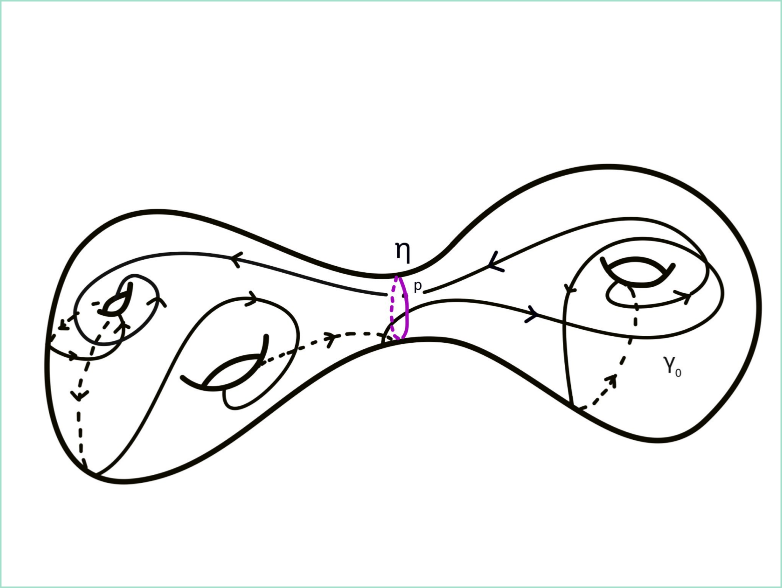

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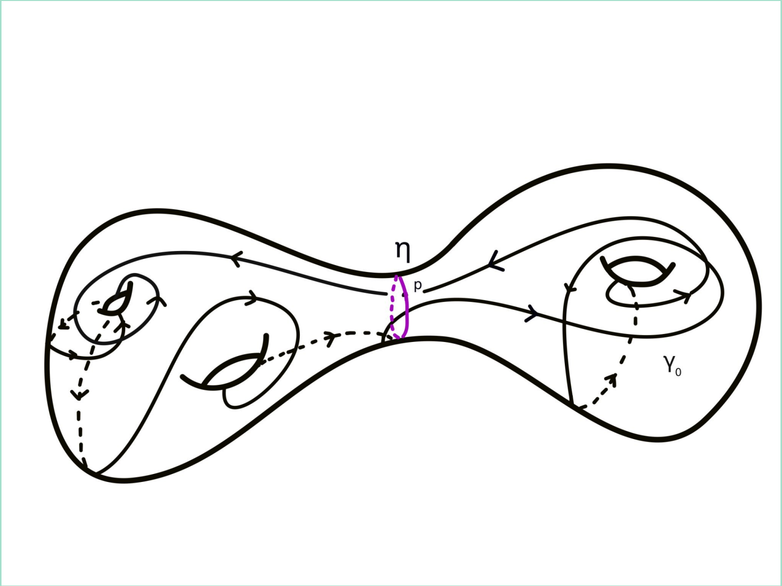
Given a pair of positive integers (m, n) , let γ be the curve $\eta^m * \gamma_0^n$ (based at the intersection point)



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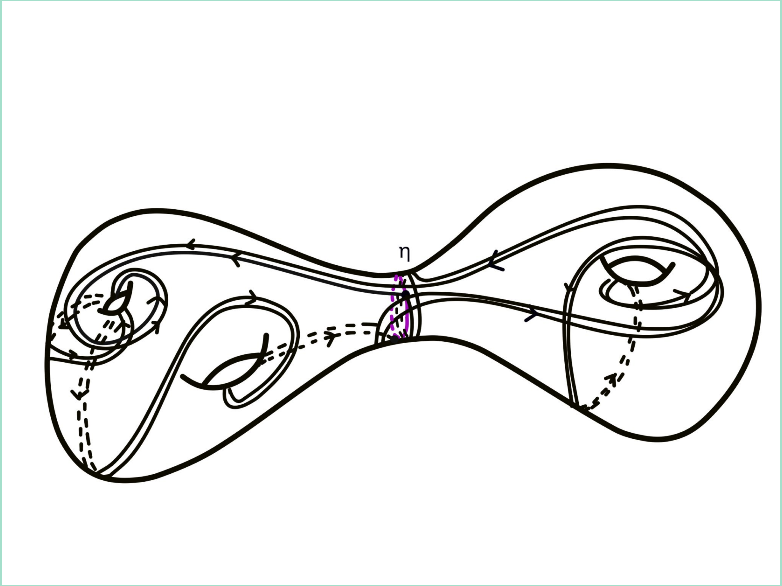
A (2,2) curve.



Are these different?

For every n, m we get a curve.

Are the different types?

How can we tell?

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A first guess...

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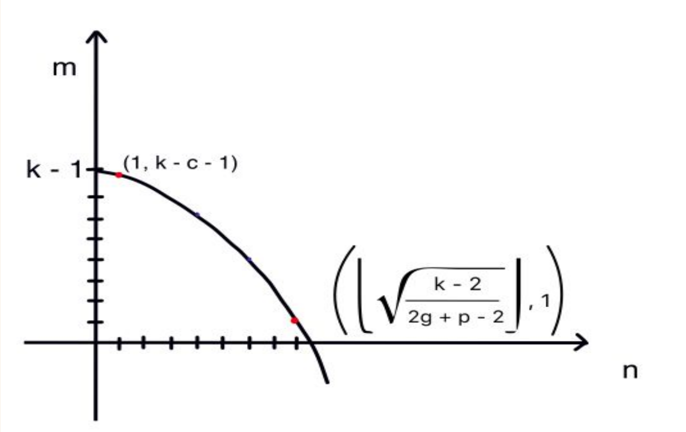
$$i(\gamma, \gamma) = i(\gamma_0, \gamma_0)n^2 + (i(\gamma_0, \eta)n - 1)m$$



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Self -Intersection number

Yes! But there are curves with same self-intersection number within the family

Solve for k

A better invariant...

For (ϕ, X) in Teich(Σ) . Let $\ell_\gamma(X)$ denote the 'X-length' of the geodesic in the free homotopy class of $\phi(\gamma)$.

We define the length infimum of γ as follows:

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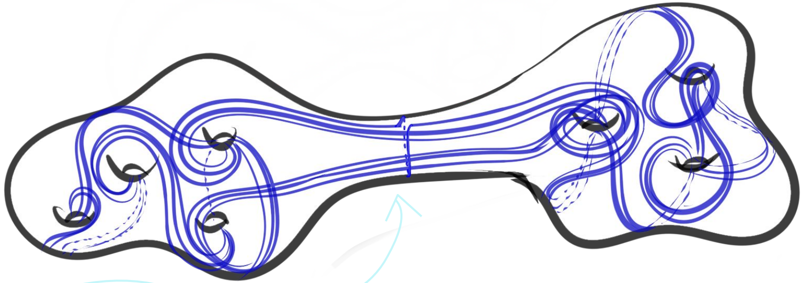
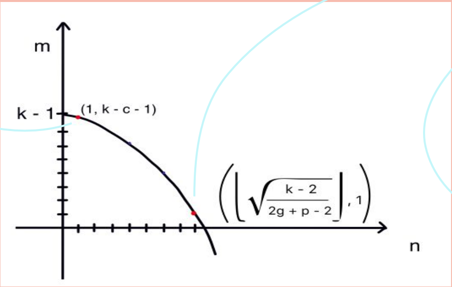
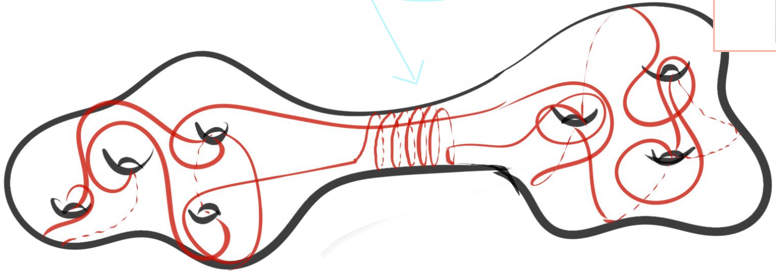
! Realized uniquely and is a MCG invariant

Back to slide 1

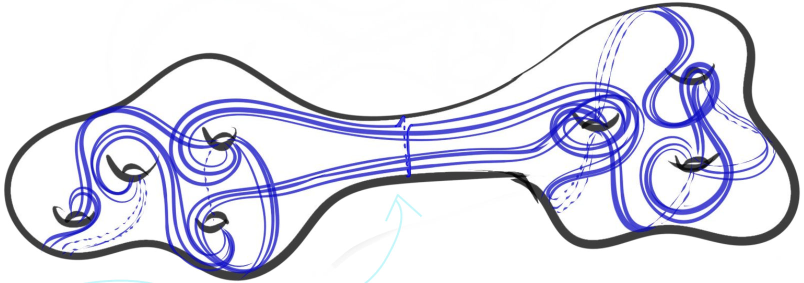


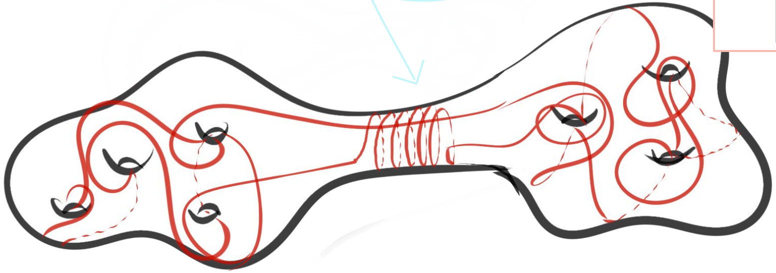


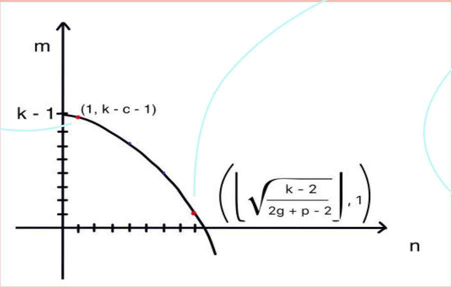


Back to slide 1

Both have same intersection number (k) ;(







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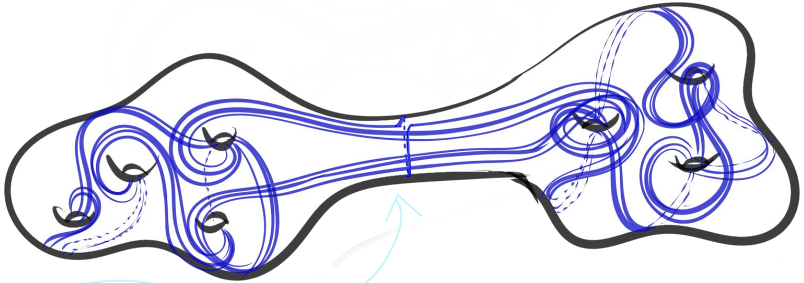
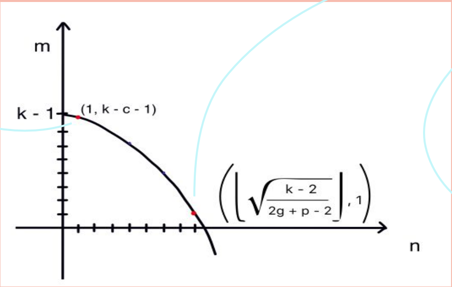
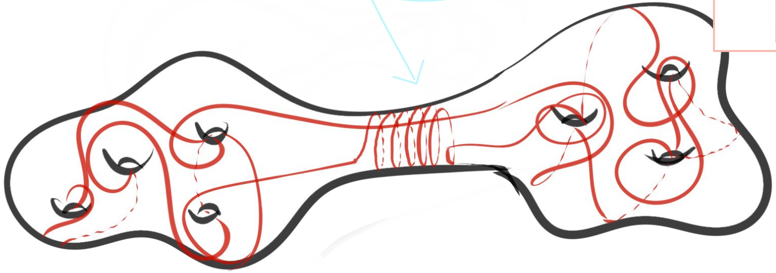




$\sqrt{k} \lesssim m_{\beta_k}$



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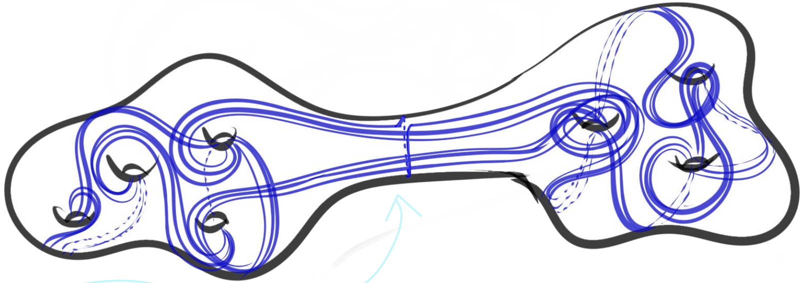

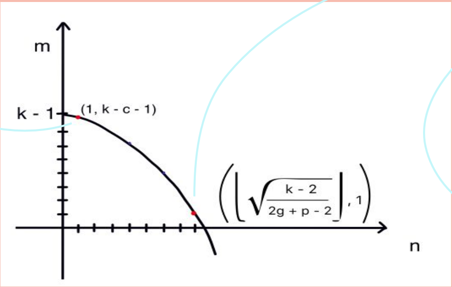

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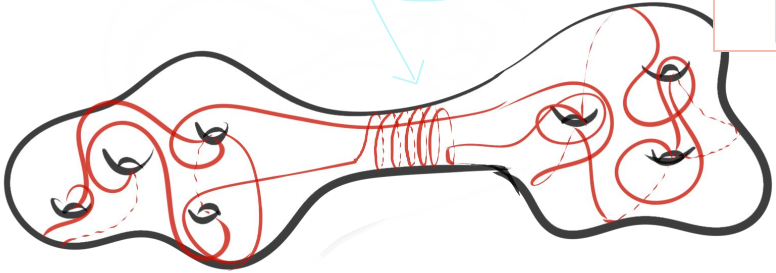

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So these are of different topological types!

Theorem (ish):

For any finite type surface and any choice of natural number k (some minor restrictions),
Can build curves with same intersection numbers (k) but of different infimum length and as a result topological types.
....and some more info on the inf metrics.

Joint work with A. Basmajian

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Thanks!

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